Traffic flow stochastic model (2×2) with discrete set of states and continuous time

Buslaev A.P., Tatashev A.G., Yashina M.V.

The present paper proposes a stochastic model of the traffic flow. This model has a discrete set of states and the continuous time. The model is a generalization of the discrete stochastis model that has been considered in a previous paper of this authors collective, for the case of a mixed traffic flow which consists of "fast" and "slow" vehicles. Moreover the continuous time model allows to developed the manner for estimation of characteristics of the traffic flow.

1. Introduction

The analysis of the traffic flow and problems of the traffic control becomes more pressing last years. Deterministic (hydrodynamic) models, that do not take into account some substantial factors as availability of several lanes, the different characteristics of traffic participants and so on, parallel with stochastic models are considered. In stochastic models the road is treated as a set of cells that proportional to the dynamic distance and a discrete time unit. The traffic flow velocity consists of the deterministic component connected with dynamic distance and a stochastic component that individual behavior into account. So it has been studied the particles of different types movement

on a set of cells. The dimension of this set is $N \times m$, $N >> 1$, $m > 1$.

The discrete model of traffic flow on a one-lane road has been considered in [1,2]. The road is represented by a set of subsequent cells. In each cell there is one or no particle (vehicle). The transitions of particles are possible only at discrete time units $n\Delta$. Suppose that a cell is occupied by a particle and the cell following ahead is vacant. Then with probability depending on the particle type (vehicle velocity) transition of the particle forwards to the following cell occurs. The steady characteristics of the considered system have been investigated. The multilane simulation models have been considered in [3]. The analysis of traffic flow characteristics on the basis of simulation has been presented in [4, 5]. Traffic flow mathematical models have been investigated in [6, 7]. A deterministic model has been investigated in [6]. It has been considered a stochastic model in terms of the queuing theory in [7].

A two-lane traffic flow discrete stochastic model, that is a generalisation of the models, considered in [1,2], has been developed in [8]. In this model a road is represented by two closed sequences of cells (lanes). The transitions of particles are possible only at discrete time units $n\Delta$. If the cell following ahead is busy and the two adjancent cells are empty then with the appropriate probability the particle is rebuilt in adjacent lane with propulsion. In [8] the approximate method has been elaborated for evaluation of the steady state probabilities of considered system fragments and such characteristics as the particle flow rate (traffic flow rate) and the changes of lane rate. The exactness of proposed approximations has been estimated by means of simulation modelling.

In the present paper the model is investigated which differs from the model considered in [8] by that in this model transitions of particles occur at arbitrary instants with an assigned intensity.

There are two types of particles characterized by the intensity of particle transitions. Thus the model takes into account the availability in the traffic flow of "fast" and "slow" vehicles.

The continuous time model is the limit case of the discrete time model when $\Delta \to 0$ and accordingly the transition probabilities $p \to 0$.

The method used for calculation of the studied characteristics is similar to the method proposed in [8]. It is proved that the continuous time model allows to deal with less complicated calculations than the appropriate discrete time model. It is explained by that *in a continuous time model every change of states of a cells set is realized by a transition of the only particle.*

2. Model description

Suppose that there are $N \times 2$ cells. Two indexes correspond to each cell. The value of the first index can be equal to $1, 2, \dots, N$ and the second index (lane index) can be equal 1 or 2. So the pair of numbers (i, j), $i = 1, 2, \dots, N$, $j = 1, 2$, corresponds to the cell. There are M_1 particles of the first type and M_2 particles of the second type. At each instant each particle occupies one cell and no cell contains more than one particle.

Suppose that the value of the first index, which equal to $N+1$, is identified with 1 and the second index, which is equal to $j+1$, is identified with 1 when $j = 2$.

The particle movement is submitted to the next rules. *If at time t a particle of the kth type* $(k = 1, 2)$ *occupies the cell* (i, j) *and the cell* $(i+1, j)$ *is vacant then during the time interval* $(t, t+\Delta)$ *with probability* $\mu_k\Delta + o(\Delta)$, $\Delta \to 0$, *the particle passes from the cell* (i, j) *to the cell* $(i+1, j)$ *(i.e. the particle move forward remaining in the same lane). If the particle of the kth type occupies the cell* (i, j), *the cell* $(i+1, j)$ *is busy, and the cells* $(i, j+1)$, $(i+1, j+1)$ *are empty then during the time interval*

$(t, t+\Delta)$ with probability $\mu_k\Delta + o(\Delta)$ the particle moves to the cell $(i+1, j+1)$, i.e. moves forward with change of lane. In the other cases the particle is not replaced. The probability of two or more transitions during the time interval is the infinitesimal of higher oder than Δ.

3. Evaluation of the steady state probabilities

Let us consider the four cells with the first index, which is equal to 1 and 2. For approximate calculation of the steady states of the four cells we use method similar to the method which has been elaborated in [8].

The mean idea of the approximated method is that the considered four cells are described by a Markov process with discrete set of state and continuous time. The transition intensity for each pair of the process is calculated provided that in every cell, adjancent to the four cells, $((N,1), (N,2), (3,1), (3,2))$ *with probability $\rho_1 = m_1/(2N)$, that is independent on the other cells states, there is a particle of the first type, with the probability $\rho_2 = m_2/(2N)$ the cell is occupied by a particle of the second type and with probability $1-\rho$ $(\rho = \rho_1 + \rho_2)$ the cell is empty.*

Taking into account that the configurations of particles, symmetrical respecting to line between the lanes, are characterized by just the same probabilities, we compound them to one state. There are 45 different states of the four cells shown in fig. 1. Suppose that equality $\theta(i,j) = k$, $k = 1, 2$, means that the cell (i,j) contains the particle of the kth type. The equality $\theta(i,j) = 0$ means that the particle (i,j) is empty. Then, for example, the states E_1 and E_2 are described as $E_1 = \{\theta(1,1) = \theta(1,2) = \theta(2,1) = \theta(2,2) = 0\}$, $E_2 = \{\theta(1,1) = \theta(1,2) = \theta(2,1) = 0,\ \theta(2,2) = 1\} \cup \{\theta(1,1) = \theta(1,2) = \theta(2,2) = 0,\ \theta(2,1) = 1\}$.

Let $P_i(t)$ be the probability that at time the state of the four cells is E_i. The considered Markov process satisfies the condi-

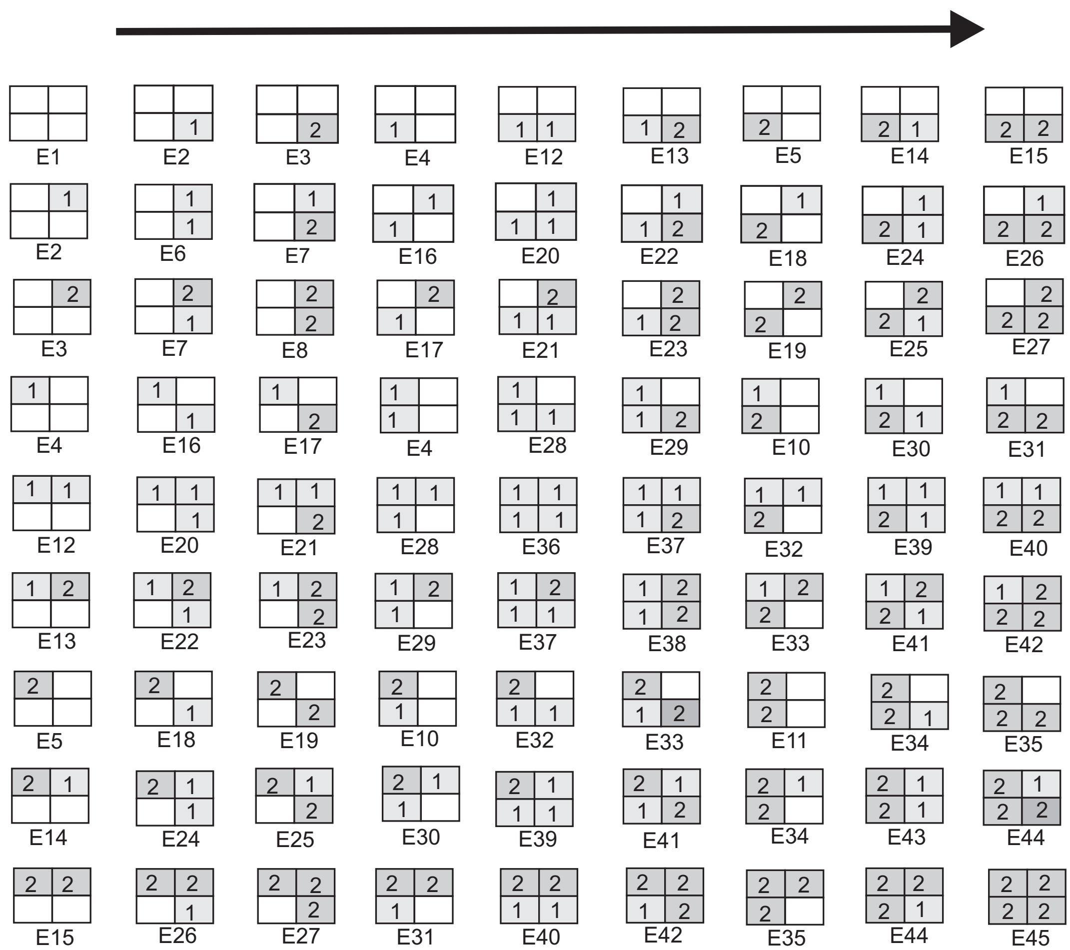

Figure 1: States of the four cells

tions of the ergodicity [9]. So the steady state probabilities p_i exist, $p_i = \lim_{t\to\infty} P_i(t)$.

Our assumptions allow to obtain the equations for the steady-state probabilities by the standard manner [9]. For example the equations corresponding to the states E_1 and E_2 are

$$-2(\rho_1\mu_1 + \rho_2\mu_2)p_1 + (1-\rho^2)\mu_1 p_2 + (1-\rho^2)\mu_2 p_3 = 0, \qquad (1)$$

$$-((1-\rho^2)\mu_1 + 2(\rho_1\mu_1 + \rho_2\mu_2))p_2 + \mu_1 p_4 + 2(1-\rho)\mu_1 p_6 + (1-\rho)\mu_2 p_7 = 0. \qquad (2)$$

The system for steady-states probabilities consists of the 45 equations, corresponding to 45 system states, and the normal-

izing equation

$$\sum_{i=1}^{45} p_i = 1. \tag{3}$$

4. Macro-characteristics of flow

According to the Markov processes theory each equations for the appropriate state is a consequence of the other ones. The system of the 45 equations and the normalizing equation has the only solution $\overline{p}_i$, $i = 1, \ldots, 4$. This solution yields the approximate values of the steady state probabilities of the four cells.

As it has been mentioned above the density ρ_k of the flow of particles of the kth type flow satisfies the relations $\rho_k = \frac{m_k}{2N}$. $k = 1, 2$. On the other hand

$$\rho_1 = \frac{1}{2}(p_4+2p_9+p_{10}+p_{12}+p_{13}+p_{16}+p_{17}+p_{20}+p_{21}+p_{22}+p_{23}+2p_{28}+ 2p_{29}+p_{30}+p_{31}+p_{32}+p_{33}+2p_{36}+2p_{37}+2p_{38}+p_{39}+p_{40}+p_{41}+p_{42}), \tag{4}$$

$$\rho_2 = \frac{1}{2}(p_5+p_{10}+2p_{11}+p_{14}+p_{15}+p_{18}+p_{19}+p_{24}+p_{25}+p_{26}+p_{27}+p_{30}+ p_{31}+p_{32}+p_{33}+2p_{34}+2p_{35}+p_{39}+p_{40}+p_{41}+p_{42}+2p_{43}+2p_{44}+2p_{45}). \tag{5}$$

Let us prove the relations (4) and (5). The pair of cells $(i, 1)$, $(i, 2)$ is identical with the pair $(j, 1)$, $(j, 2)$, $1 \le i, j \le N$. So the mean number of particles of kth type ($k = 1, 2$) in each pair equals M_k/N, i.e. equals $2\rho_k$. On the other hand the mean number of the particles of the kth type in the pair of cells is equal to the sum of probabilities of the states of the considerd four cells in that there is exactly one particle in the pair of cells and the double probabilities of the states with two particles in this pair of cells. Hence the equations (4) and (5) are true.

Let $\overline{\rho}_k$ be the value of ρ_k which satisfies the relations (4) and (5) if in this relations the approximate values $\overline{p}_i$ substitute for exact values p_i.

Let mean number q_k of particles of the kth type ($k = 1, 2$) that leave the pair of cells (1,1), (1,2) be called the particles of kth type flow intensity and mean number h_k of particles of the kth type leaving the pair cells with a change of lane be called the changes intensity of the particles of the kth type.

There is one particle, which can move, in the pair of cells (1,1) and (1,2), when the state of the four cells is one from the states E_4, E_{10}, E_{12}, E_{13}, E_{16}, E_{17}, E_{28}, E_{29}, E_{30}, E_{31}. If the state of the four cells is E_9 then there are two particles which can move. Hence the value of q_1 can be calculated as

$$q_1 = (p_4+2p_9+p_{10}+p_{12}+p_{13}+p_{16}+p_{17}+p_{28}+p_{29}+p_{30}+p_{31})\mu_1. \tag{6}$$

Similarly we obtain the relations

$$h_1 = (p_{12} + p_{13})\mu_1, \tag{7}$$

$$q_2 = (p_5+p_{10}+2p_{11}+p_{14}+p_{15}+p_{18}+p_{19}+p_{32}+p_{33}+p_{34}+p_{35})\mu_2, \tag{8}$$

$$h_2 = (p_{14} + p_{15})\mu_2. \tag{9}$$

Let $\overline{q}_k$, $\overline{h}_k$ ($k = 1, 2$) be values of q_k and h_k, which are obtained if in equations (6)–(9) p_i substitutes for $\overline{p}_i$.

If $\mu_1 > \mu_2$ then $\bar{q}_1$ is some greater than the value q_1 obtained by simulation and in contrast $\bar{q}_2$ is some smaller then the value q_2 obtained by simulation. As it be shown below the difference between the calculated approximate values and data obtained by simulation is smaller if values $\tilde{q}_k$, $\tilde{h}_k$ determined by means of the correcting coefficient substitute for values q_k, h_k

$$\tilde{q}_k = (\rho_k/\bar{\rho}_k)\bar{q}_k,\ \ k = 1, 2, \tag{10}$$

$$\tilde{h}_k = (\rho_k/\bar{\rho}_k)\overline{h}_k,\ \ k = 1, 2. \tag{11}$$

Let us explain why the correcting coefficient improves he approximation. Let the ratio of the mean number of transitions of

particles of the kth type to number of the particles of this type be called the velocity of the particles of the kth type. Let the velocity of particles of the kth type be v_k. Then

$$v_k = q_k/(2\rho_k),\ k = 1, 2. \tag{12}$$

Because of the relation $q_k = 2\rho_k \cdot v_k$ the obtained approximate value of the particles of the kth type differs from the exact value by the coefficient $\bar{q}_2/\rho$. Coefficient $\rho/\bar{q}_2$ compensates the error.

Suppose

$$\bar{v}_k = \bar{q}_k/(2\rho_k), \tag{13}$$

$$\tilde{v}_k = \tilde{q}_k/(2\rho_k). \tag{14}$$

5. Approximations by means of Bernoulli scheme

Let us consider another manner of evaluation of the steady-state probabilities and the macro-characteristics that is more simple than the manner described above. Assume that the probability of the state of each cell is independent of the states of other cells, and the cells with probability ρ_1 contains a particle of the first type, with probability ρ_2 contains a particle of the second type and with probability $1-\rho$ the cell is empty. Let $\hat{p}_i$ $(i = 1, 2, \dots 45)$, $\hat{v}_k$, $\hat{q}_k$, $\hat{h}_k$ $(k = 1, 2)$ be values of the appropriate characteristics that are obtained when these assumptions are used.

Then $\hat{p}_1 = (1-\rho)^4$, $\hat{p}_2 = \hat{p}_4 = 2\rho_1(1-\rho)^3$, $\hat{p}_3 = \hat{p}_5 = 2\rho_2(1-\rho)^3$, $\hat{p}_6 = \hat{p}_9 = \rho_1^2(1-\rho)^2$, $\hat{p}_7 = \hat{p}_{10} = \hat{p}_{13} = \hat{p}_{14} = \hat{p}_{17} = \hat{p}_{18} = 2\rho_1\rho_2(1-\rho)^2$, $\hat{p}_8 = \hat{p}_{11} = \rho_2^2(1-\rho)^2$, $\hat{p}_{12} = \hat{p}_{16} = 2\rho_1^2(1-\rho)^2$, $\hat{p}_{15} = \hat{p}_{19} = 2\rho_2^2(1-\rho)^2$, $\hat{p}_{20} = \hat{p}_{28} = 2\rho_1^3(1-\rho)$, $\hat{p}_{21} = \hat{p}_{22} = \hat{p}_{24} = \hat{p}_{29} = \hat{p}_{30} = \hat{p}_{32} = 2\rho_1^2\rho_2(1-\rho)$, $\hat{p}_{23} = \hat{p}_{25} = \hat{p}_{26} = \hat{p}_{31} = \hat{p}_{33} = \hat{p}_{34} = 2\rho_1\rho_2^2(1-\rho)$, $\hat{p}_{27} = \hat{p}_{35} = 2\rho_2^3(1-\rho)$, $\hat{p}_{36} = \rho_1^4$, $\hat{p}_{37} = \hat{p}_{39} = 2\rho_1^3\rho_2$, $\hat{p}_{38} = \hat{p}_{43} = \rho_1^2\rho_2^2$, $\hat{p}_{40} = \hat{p}_{41} = 2\rho_1^2\rho_2^2$, $\hat{p}_{42} = \hat{p}_{44} = 2\rho_1\rho_2^3$, $\hat{p}_{45} = \rho_2^4$; $\hat{v}_k = (1-\rho)(1+\rho-\rho^2)\mu_k,\ k = 1, 2,$

$\hat{q}_k = 2\rho_k(1-\rho)(1+\rho-\rho^2)\mu_k,\ \ k=1,2,\ \hat{h}_k = 2\rho_k\rho(1-\rho)^2\mu_k,$ $k=1,2.$

6. Sistem solution and results of calculation

Suppose $\rho_1 = \rho_2 = 0.25$.

The calculated approximate values are represented in table 1. These values are compared to values obtained by simulation. The symbols for values which have been obtained by simulations are marked by star. The duration of the simulation interval is equal to 12000 time units, $N = 500$.

We do not present the tables of calculations for $\rho_1 = 0.54$, $\rho_2 = 0.06$ and $\rho_1 = 0.08$, $\rho_2 = 0.72$.

The dependence of state probabilities on state numbers is represented on fig 2-10.

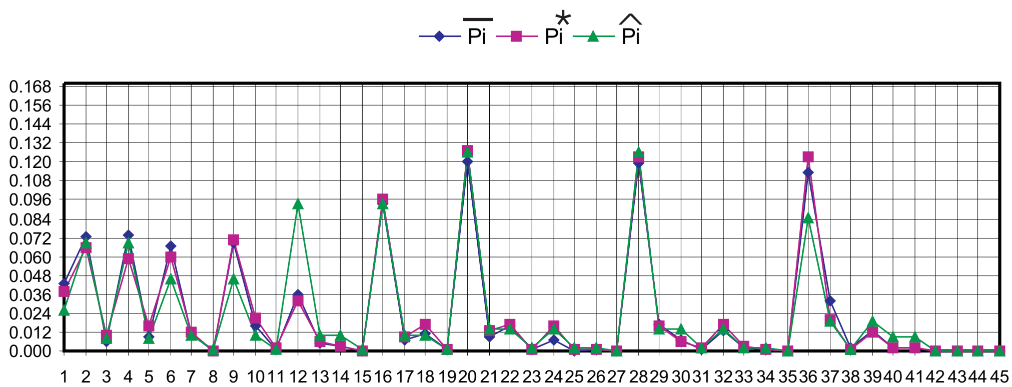

Figure 2: State probabilities for $\rho_1 = \rho_2 = 0.25$, $\mu_1 = 2$, $\mu_2 = 1$

The calculated values of macro-characteristics are represented in table 2.

7. Analysis of approximation accuracy

Let us compare the values $\tilde{q}_k$, $\tilde{v}_k$, $\tilde{h}_k$, $\bar{q}_k$, $\bar{v}_k$, $\bar{h}_k$, $\hat{q}_k$, $\hat{v}_k$, $\hat{h}_k$ with the values obtained by simulation.

Table 1: State probabilities, $\rho_1 = \rho_2 = 0.25$

	$\mu_1 = 2,\ \mu_2 = 1$			$\mu_1 = 3,\ \mu_2 = 1$			$\mu_1 = 4,\ \mu_2 = 1$		
i	$\bar{p}_i$	p_i^*	$\hat{p}_i$	$\bar{p}_i$	p_i^*	$\hat{p}_i$	$\bar{p}_i$	p_i^*	$\hat{p}_i$
1	0.091	0.098	0.063	0.094	0.005	0.063	0.097	0.133	0.063
2	0.064	0.049	0.063	0.067	0.038	0.063	0.069	0.037	0.063
3	0.053	0.072	0.063	0.050	0.069	0.063	0.047	0.082	0.063
4	0.061	0.032	0.063	0.061	0.019	0.063	0.062	0.014	0.063
5	0.064	0.087	0.063	0.066	0.090	0.063	0.067	0.099	0.063
6	0.026	0.023	0.016	0.028	0.021	0.016	0.029	0.022	0.016
7	0.041	0.040	0.031	0.040	0.037	0.031	0.039	0.035	0.031
8	0.014	0.019	0.016	0.011	0.015	0.016	0.008	0.015	0.016
9	0.022	0.011	0.016	0.021	0.006	0.016	0.021	0.004	0.016
10	0.045	0.042	0.031	0.045	0.035	0.031	0.045	0.027	0.031
11	0.023	0.032	0.016	0.023	0.039	0.016	0.023	0.038	0.016
12	0.012	0.010	0.031	0.013	0.008	0.031	0.013	0.007	0.031
13	0.015	0.014	0.031	0.018	0.013	0.031	0.019	0.012	0.031
14	0.008	0.008	0.031	0.006	0.006	0.031	0.005	0.005	0.031
15	0.011	0.012	0.031	0.011	0.011	0.031	0.010	0.012	0.031
16	0.033	0.023	0.031	0.033	0.017	0.031	0.033	0.013	0.031
17	0.025	0.023	0.031	0.021	0.015	0.031	0.019	0.012	0.031
18	0.034	0.036	0.031	0.035	0.036	0.031	0.036	0.031	0.031
19	0.025	0.037	0.031	0.021	0.037	0.031	0.018	0.037	0.031
20	0.023	0.025	0.016	0.021	0.026	0.016	0.021	0.027	0.016
21	0.014	0.015	0.016	0.014	0.015	0.016	0.013	0.015	0.016
22	0.025	0.023	0.016	0.032	0.027	0.016	0.036	0.027	0.016
23	0.018	0.017	0.016	0.018	0.018	0.016	0.017	0.017	0.016
24	0.010	0.014	0.016	0.007	0.014	0.016	0.006	0.010	0.016
25	0.007	0.010	0.016	0.005	0.008	0.016	0.005	0.005	0.016
26	0.013	0.012	0.016	0.012	0.011	0.016	0.011	0.008	0.016
27	0.009	0.011	0.016	0.006	0.007	0.016	0.005	0.007	0.016
28	0.016	0.014	0.016	0.016	0.016	0.016	0.016	0.015	0.016
29	0.023	0.018	0.016	0.027	0.018	0.016	0.030	0.015	0.016
30	0.009	0.009	0.016	0.006	0.008	0.016	0.005	0.005	0.016
31	0.012	0.009	0.016	0.011	0.008	0.016	0.010	0.008	0.016
32	0.017	0.021	0.016	0.018	0.026	0.016	0.018	0.025	0.016
33	0.024	0.023	0.016	0.029	0.033	0.016	0.032	0.036	0.016
34	0.009	0.008	0.016	0.006	0.010	0.016	0.005	0.006	0.016
35	0.013	0.015	0.016	0.011	0.013	0.016	0.010	0.012	0.016
36	0.007	0.009	0.004	0.008	0.019	0.004	0.008	0.027	0.004
37	0.020	0.017	0.008	0.026	0.028	0.008	0.030	0.032	0.008
38	0.013	0.008	0.004	0.020	0.014	0.004	0.025	0.015	0.004
39	0.007	0.014	0.008	0.005	0.017	0.008	0.004	0.016	0.008
40	0.010	0.009	0.008	0.009	0.013	0.008	0.009	0.012	0.008
41	0.010	0.012	0.008	0.009	0.012	0.008	0.007	0.013	0.008
42	0.014	0.008	0.008	0.014	0.011	0.008	0.014	0.008	0.008
43	0.002	0.003	0.004	0.001	0.004	0.004	0.001	0.003	0.004
44	0.005	0.005	0.008	0.003	0.004	0.008	0.002	0.002	0.008
45	0.003	0.003	0.004	0.002	0.002	0.004	0.002	0.002	0.004

Table 2: Macro-characteristics (the error that is relative to values obtained by simulation

	$\mu_1 = 2,\ \mu_2 = 1$		$\mu_1 = 3,\ \mu_2 = 1$		$\mu_1 = 4,\ \mu_2 = 1$	
k	1	2	1	2	1	2
$\rho_1 = \rho_2 = 0.25$						
q_k^*	0.432	0.353	0.507	0.375	0.544	0.366
$\tilde{q}_k$	0.512 (+19)	0.337(−5)	0.717(+42)	0.361 (−4)	0.923 (+70)	0.372(+2)
$\hat{q}_k$	0.625 (+45)	0.313(−12)	0.938(+85)	0.313 (−17)	1.250(+130)	0.313(−15)
$\bar{q}_k$	0.590 (+37)	0.296(−16)	0.879(+74)	0.294 (−22)	1.176(+116)	0.292(−20)
h_k^*	0.048	0.020	0.063	0.017	0.076	0.017
$\tilde{h}_k$	0.047 (−2)	0.022 (+8)	0.076(+20)	0.021 (+23)	0.101(+32)	0.019(+12)
$\hat{h}_k$	0.125 (+160)	0.063(+212)	0.188(+198)	0.063(+268)	0.250(+229)	0.063(+268)
$\bar{h}_k$	0.054 (+13)	0.019 (−5)	0.093(+47)	0.017 (0)	0.128 (+69)	0.015(−12)
v_k^*	0.864	0.706	1.014	0.750	1.088	0.732
$\tilde{v}_k$	1.024 (+19)	0.674 (−5)	1.434 (+42)	0.722(−4)	1.846 (+70)	0.743 (+2)
$\hat{v}_k$	1.250 (+45)	0.625 (−12)	1.875 (+85)	0.625(−17)	2.500 (+130)	0.625 (−15)
$\bar{v}_k$	1.180 (+37)	0.592 (−16)	1.758 (+74)	0.588(−22)	2.352 (+116)	0.584(−20)
$\rho_1 = 0.54,\ \rho_2 = 0.06$						
q_k^*	1.024	0.083	1.335	0.092	1.624	0.101
$\tilde{q}_k$	0.989 (-3)	0.076(−8)	1.465 (+10)	0.084(−8)	1.944(+20)	0.090(−11)
$\hat{q}_k$	1.071 (+4)	0.060(−28)	1.607(+20)	0.060(−35)	2.143(+32)	0.060(−41)
$\bar{q}_k$	1.026(+19)	0.058(−30)	1.530(+15)	0.057(−38)	2.048(+26)	0.056(−45)
h_k^*	0.076	0.003	0.099	0.002	0.128	0.001
$\tilde{h}_k$	0.079(+4)	0.004(+30)	0.121(+22)	0.003 (+48)	0.159(+24)	0.003(+220)
$\hat{h}_k$	0.207(+172)	0.012(+284)	0.311(+214)	0.012 (+476)	0.415(+224)	0.012(+1052)
$\bar{h}_k$	0.082(+8)	0.003 (0)	0.126(+27)	0.002 (0)	0.168(+31)	0.002(+100)
v_k^*	0.948	0.692	1.236	0.767	1.504	0.842
$\tilde{v}_k$	0.915(−3)	0.630(−8)	1.356 (+10)	0.704(−8)	1.800(+20)	0.747(−11)
$\hat{v}_k$	0.992(+4)	0.496(−28)	1.488(+20)	0.496(−35)	1.984(+32)	0.496(−41)
$\bar{v}_k$	0.950(+19)	0.483(−30)	1.417(+15)	0.475(−38)	1.896(+26)	0.467(−45)
$\rho_1 = 0.08,\ \rho_2 = 0.72$						
q_k^*	0.044	0.334	0.039	0.344	0.048	0.346
$\tilde{q}_k$	0.045(+2)	0.351(+5)	0.050(+28)	0.371(+8)	0.056(+17)	0.374(+8)
$\hat{q}_k$	0.074(+69)	0.334(0)	0.111(+186)	0.334(−3)	0.149(+210)	0.334(−3)
$\bar{q}_k$	0.072(+64)	0.328(−2)	0.105(+169)	0.325(−5)	0.140(+192)	0.305(−12)
h_k^*	0.002	0.015	0.003	0.017	0.004	0.016
$\tilde{h}_k$	0.004(+87)	0.020(+36)	0.004(+43)	0.023(+34)	0.006(+60)	0.025(+54)
$\hat{h}_k$	0.010(+412)	0.046(+207)	0.015(+412)	0.046(+171)	0.021(+412)	0.046(+188)
$\bar{h}_k$	0.006(+200)	0.019(+27)	0.009(+200)	0.020(+18)	0.016(+300)	0.020(+25)
v_k^*	0.275	0.232	0.244	0.239	0.300	0.240
$\tilde{v}_k$	0.280 (+2)	0.244 (+5)	0.313(+28)	0.258(+8)	0.350(+17)	0.260(+8)
$\hat{v}_k$	0.464 (+69)	0.232 (0)	0.696(+186)	0.232(−3)	0.928(+210)	0.232(−3)
$\bar{v}_k$	0.450 (+64)	0.228 (−2)	0.656(+169)	0.226(−5)	0.875(+192)	0.212(−12)

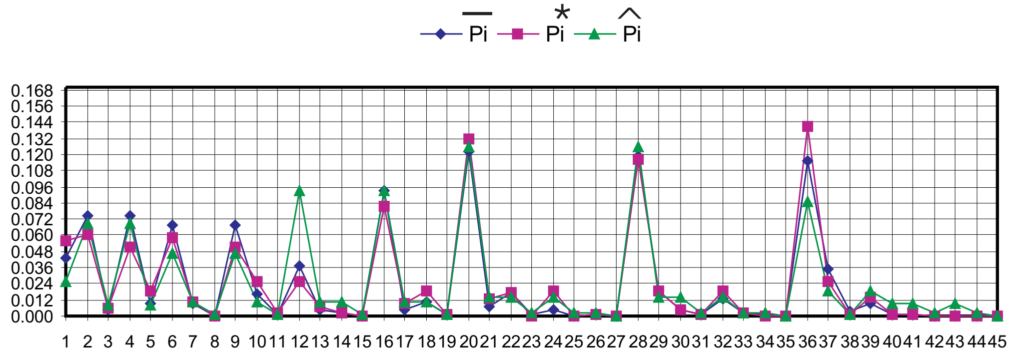

Figure 3: State probabilities for $\rho_1 = \rho_2 = 0.25$, $\mu_1 = 3$, $\mu_2 = 1$

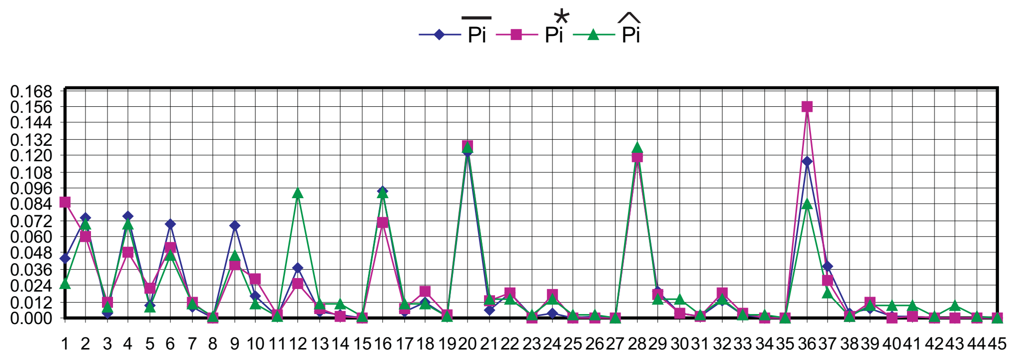

Figure 4: State probabilities for $\rho_1 = \rho_2 = 0.25$, $\mu_1 = 4$, $\mu_2 = 1$

Suppose $\rho_1 = \rho_2 = 0.25$. The values $\tilde{q}_1$ $(\tilde{v}_1)$ exceeds the values q_1^* (v_1^*). The relative error makes up the values from 19% to 70% on the three considered set of data. The value $\bar{q}_1$ $(\bar{v}_1)$ exceeds the value q_1^* (v_1^*). The difference $\bar{q}_1$ and q_1^* makes up from 37% to 116% of q_1^*. The value $\hat{q}_1$ $(\hat{v}_1)$ exceeds the value q_1^* (v_1^*). The relative error make up from 45% to 113%.

In the other cases the approximate values calculated according to formulas (10), (11), (14) are also nearer to value obtained by simulation than the values calculated by the two others manners, or in some cases the formulas (10), (11), (14) yields approximation only few worse.

In each the three cases the value of $\tilde{q}_2$ $(\tilde{v}_2)$ difference the values

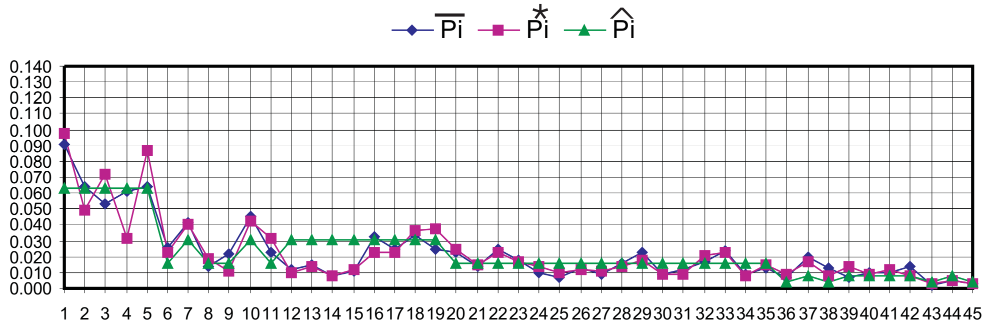

Figure 5: State probabilities for $\rho_1 = 0.54$, $\rho_2 = 0.06$, $\mu_1 = 2$, $\mu_2 = 1$

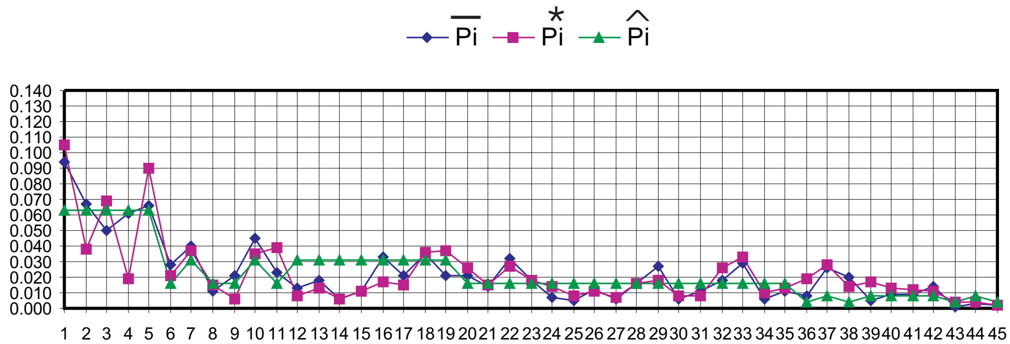

Figure 6: State probabilities for $\rho_1 = 0.54$, $\rho_2 = 0.06$, $\mu_1 = 3$, $\mu_2 = 1$

q_2^* (v_2^*) is not greater than 5% of q_2^*. The value $\tilde{h}_1$ difference from the value h_1^* not greater than 32% of h_1^*. The value h_2^* exeeds h_2^*. The relative error is not greater than 23%.

Usually the relative error is the better the smaller is the ratio of μ_1 to μ_2 $(\mu_1 > \mu_2)$.

The values $\tilde{p}_1, \tilde{p}_2, \ldots, \tilde{p}_{45}$ that satisfies the system of linear equations usually differs from the values obtained simulation less than values calculated by means of Bernoulli scheme.

The analysis of calculation for the cases $\rho_1 = 0.54$, $\rho_2 = 0.06$, and $\rho_2 = 0.06$, $\rho_1 = 0.72$, also shows that meaning $\tilde{v}_k$, $\tilde{q}_k$, $\tilde{h}_k$, are usually more close differs from the values obtained by simulation less than values $\bar{v}_k$, $\bar{q}_k$, $\bar{h}_k$, $\hat{v}_k$, $\hat{q}_k$, $\hat{h}_k$.

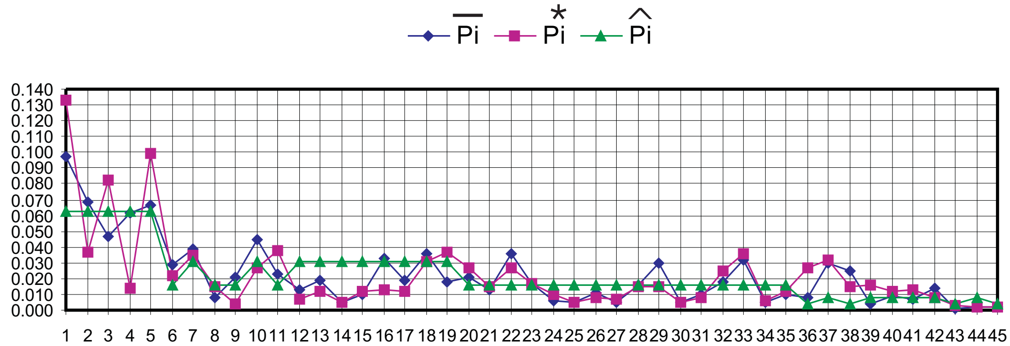

Figure 7: State probabilities for $\rho_1 = 0.54$, $\rho_2 = 0.06$, $\mu_1 = 4$, $\mu_2 = 1$

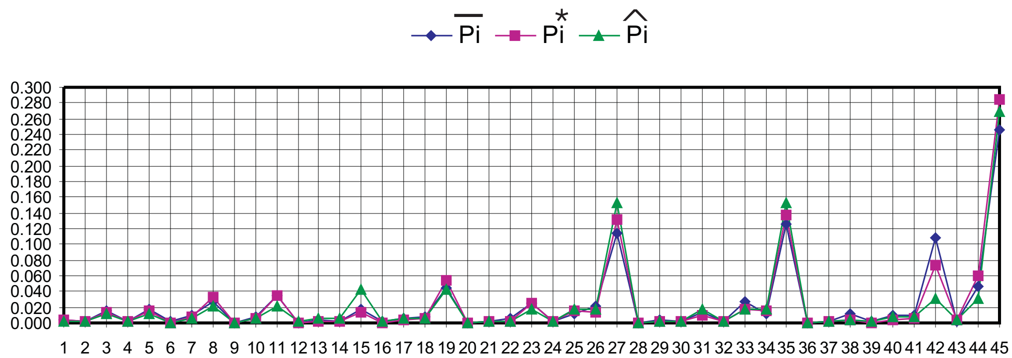

Figure 8: State probabilities for $\rho_1 = 0.08$, $\rho_2 = 0.72$, $\mu_1 = 2$, $\mu_2 = 1$

For $\rho_1 = 0.54$, $\rho_2 = 0.06$, the largest difference between $\tilde{q}_k$ ($\tilde{v}_k$) and q_k^* (v_k^*) corresponds to the case $k = 1$, $\mu_1 = 4$, $\mu_2 = 1$. In this case the value $\tilde{q}_1$ exceeds the value q_2^*. The relative error is not greater than 20% .

For $\mu_1 = 0.08$, $\mu_2 = 0.72$ the largest difference between $\tilde{q}_k$ ($\tilde{v}_k$) q_k^* (v_k^*) is reached in the case $k = 1$, $\mu_1 = 3$, $\mu_2 = 1$. In this case the value $\tilde{q}_1$ ($\tilde{v}_1^*$) exceeds values q_1^* (v_1^*). The relative error is not greater than 28%.

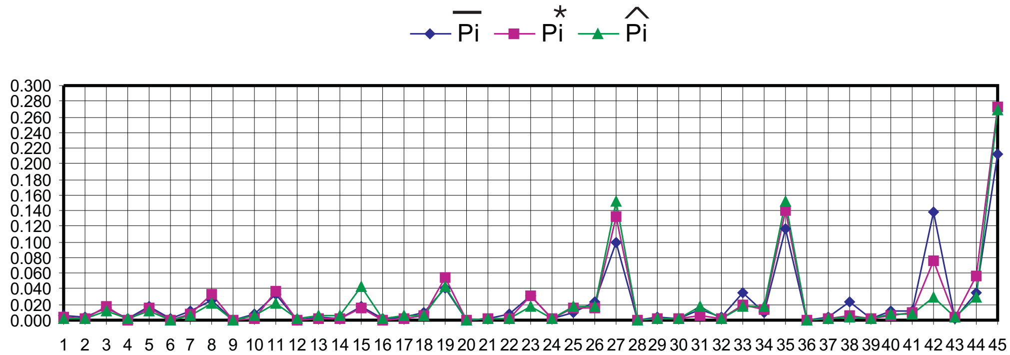

Figure 9: State probabilities for $\rho_1 = 0.08$, $\rho_2 = 0.72$, $\mu_1 = 3$, $\mu_2 = 1$

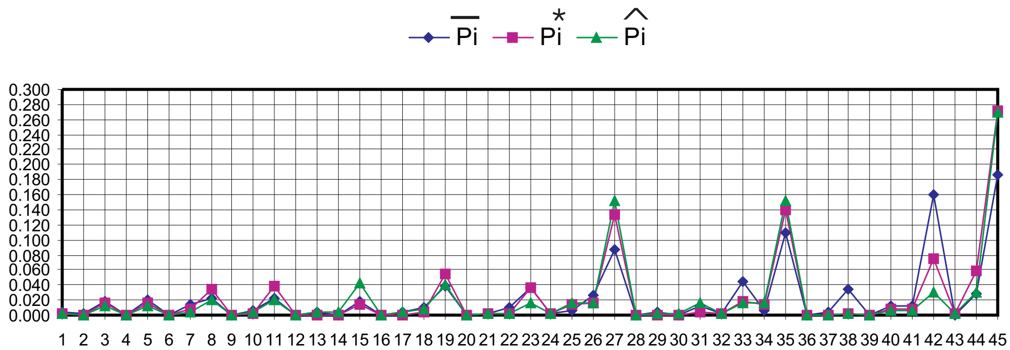

Figure 10: State probabilities for $\rho_1 = 0.08$, $\rho_2 = 0.72$, $\mu_1 = 4$, $\mu_2 = 1$

8. Analysis of results of calculations of macro-characteristics

If Bernoulli scheme approximation is used then calculated values of macro-characteristics are proportional to the particle transitions intensity.

On the other hand it is clear intuitively that the fast particles occur more often than slow ones in the states in that their transitions are prevented by other particles. Hence the ratio of velocity (v_k) of particle to its transition intensity (μ_k) for the fast particles is less than for the slow particles. It is confirmed both by simulation and calculations. The difference between v_2/μ_2

and v_1/μ_1 is the greater the greater the μ_1/μ_2 is. Because of that for fixed μ_2 both value v^* and value $\tilde{v}_1$ calculated according to formulas (10), (14) increases slower than linearly if μ_1 increases. But it can be noticed that for the valued obtained by simulation the slowing down of increasing of the velocity of the particle of the first type with increasing of μ_1 is more obvious. It can be noticed also if μ_2 is fixed and μ_1 increases then the values $\bar{v}_2$ and v_2^* increases slowly and the value $\bar{v}_2$ is constant. For fixed μ_2 and increasing μ_1 the values $\bar{h}_1$ and h_1^* increase approximately linearly. The ratio of $\bar{h}_k/\bar{q}_k$ to h_k^*/q_k^* for the fast particles is greater than for slow ones (moving forward the fast particles change lane more often than slow particles).

9. Conclusion

It is presented a model of the traffic flow on two lanes. An approximate manner is elaborated for calculation the steady state probabilities and macro-characteristics of model. The results of calculations by means of the elaborated method are compared with results of simulation. This comparison shows that the presented method yield rather good approximation and describes the qualitative behavior of the considered characteristics.

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